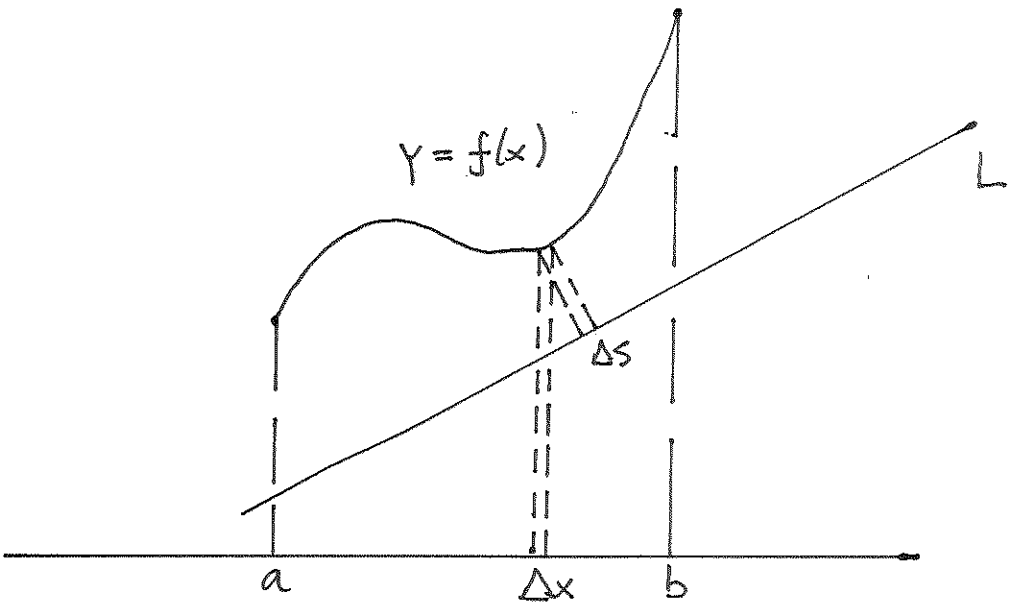
Volume of Revolution



– if $y = f(x)$ is a graph with respect to the line $L : y = mx + n$ then $1 + mf'(x) \neq 0$

– let V be the volume generated by rotating the region R bounded by the curve $y = f(x)$ as a graph over the line L

– relation between Δs and Δx:

$$\Delta s = \text{projection of the vector } (\Delta x, \Delta y) \text{ in the direction } (\cos\theta, \sin\theta)$$

$$= \left| \left(1, \frac{\Delta y}{\Delta x}\right) \cdot (\cos\theta, \sin\theta) \right| \Delta x$$

$$= \frac{1}{\sqrt{1+m^2}} |1 + mf'(x)| \, \Delta x$$

$$\Longrightarrow \qquad V = \frac{\pi}{(1+m^2)^{\frac{3}{2}}} \int_a^b (f(x) - mx - n)^2 |1 + mf'(x)| dx$$